AN EVALUATION OF THE TRW METHOD OF COMBINING POINT ESTIMATES OF RELIABILITY

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THESIS

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by

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by

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ABSTRACT

A computer program was written that carries out reliability assessments according to a method proposed by the TRW Corporation. This method combines point estimates for reliability from different sources into an overall point estimate. The program was used to calculate the overall point estimate for cases covering a range of sample sizes, and underlying probabilities of success in order to make a judgement on the usefulness of the method.



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I. INTRODUCTION

In deriving a point estimate for the reliability of a system or component, there is no universally accepted method for combining data from more than one test environment. For example, if a system on a test stand performs its intended function four times out of five trials, and in another environment, say actual use, it performs its function in two out of three trials, the point estimates for system reliability in the two environments would be 4/5 and 2/3 respectively. It is not clear how these two estimates from different environments should be combined into a single point estimate of realiability.

The Quality Evaluation Laboratory, U.S. Naval Ammunition
Depot, Oahu, Hawaii, published a procedure to accomplish this
integration of trial data by comparing all data sets to a
base data set and generating weighting factors for each data
set. TRW Corporation wrote a computer routine to carry out
the required computations for this method and proposed the
method be adopted use in certain development programs.

The TRW computer program was not available for this study and so two programs were written as part of this project. Each program has certain limitations in use, but together they cover a complete range of method parameters.



II. THE METHOD

Consider a series of n trials of a system of which m are judged successes. Then the probability that R is the underlying reliability of the system is given by the binomial distribution.

$$P(R) = {}_{n}C_{m} R^{m} (1-R)^{n-m}, (0 \le R \le 1).$$

This relationship generates a probability density function such as is shown in figure 1. This is the likelihood function of R given m successes in n trials.

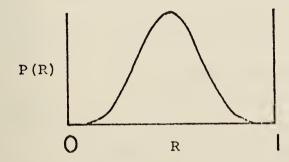


FIGURE 1. Likelihood function of R given m successes in n trials.

To evaluate percentiles of the binomial distribution, the incomplete beta function can be used through the following identity:

$$I_{R}(a,n-a+1) = \sum_{x=a}^{n} {}_{n}C_{x}R^{x}(1-R)^{n-x}$$
 , n and k positive integers.

 $^{
m l}$ The incomplete beta function is defined

$$I_{x}(a,b) = \frac{1}{B(a,b)} \int_{0}^{x} t^{a-1} (1-t)^{b-1} dt, (0 \le t \le 1)$$

where

$$B(a,b) = \frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)}$$

The incomplete beta family is extensively tabled and efficient computer routines have been written to compute I_x (a,b).



If the probability density function for a second set of test data is superimposed on the first density function, a representation such as figure 2 results.

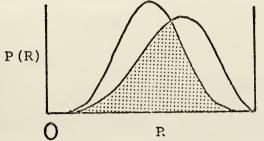


FIGURE 2. Two likelihood functions showing overlap.

The TRW method uses the amount of overlap as a weighting factor that measures the extent of agreement between the two sets of data. Calculating the weighting factor can be done by first solving for points of intersection of the two curves. These are all points x which are solutions to the equation

$$_{n}^{C}C_{m}^{m}(1-x)^{n-m} = _{p}^{C}C_{s} x^{s}(1-x)^{p-s}$$

where

n = number of trials in data set 1

m = number of successes in data set 1

p = number of trials in data set 2

s = number of successes in data set 2

This expression simplifies to

$$x^{m-s}(1-x)^{n-m-p+s} = \frac{p! (n-m)! m!}{(p-s)! s! n!}$$

This is a polynomial of degree n-p.

With the points of intersection determined, the TRW method determines the weighting factor by summing the area under that curve which is the lower between each intersection. See figure 2.



The two computer programs included in this report perform these calculations. Limitations on the programs are discussed in the descriptions that introduce each program.

To calculate a point estimate of reliability using the TRW method, each set of data, $i=1,2,\ldots,n$, is compared with a set of base data and a weighting factor, k_i , is determined as discussed above. These weighting factors are then applied to the point estimates to determine the overall estimate as follows:

$$R = \frac{1.0(P_b') + k_1(P_1') + k_2(P_2') + \dots + k_n(P_n')}{1.0 + k_1 + k_2 \dots + k_n}$$

where

R = overall point estimate of reliability

P; = point estimate from ith data set

 P_{b}' = point estimate from base data set

k; = weighting factor from ith data set

i = 1, 2, ..., n



III. ANALYSIS OF TRW METHOD

The magnitude of the weighting factor or overlap used in this method is determined by the relationship between the two incomplete beta functions used to represent the data sets. Specifically, the overlap is a function of (1) the location of the mean of each curve relative to the mean of the other end and (2) the spread or variance of each curve. A particular realization of the beta function tends to become spike shaped as the number of trials is increased. This effect is shown in table 1 which lists by way of example seven data sets, all having a mean of .8000.

Number of trials	Beta curve	Variance
9	D/ 5 1)	0246
18	B(x;5,4) B(x;10,8)	.0246 .0129
25	B(x; 20, 5)	.0062
40	B(x; 32,8)	.0039
50	B(x;40,10)	.0031
60	B(x; 48, 12)	.0026
100	B(x;80,20)	.0016

Table 1. As the number of trials is increased, the variance of the beta distribution representation decreases. The means of all data sets is .8000.

It was hypothesized that the weighting factor would be relatively small (and consequently the TRW method would have only

mean =
$$\frac{s}{(s+f)}$$
 variance = $\frac{s \times f}{(s+f)^2 \times (s+f+1)}$.

The mean and variance of the beta distribution representing a data set with s successes and f failures are



a minor effect on reliability calculations) when (1) the ratios of successes to total trials from each data set were widely displaced from each other, or (2) when the number of trials in either data set was not small and the ratio of successes to number of trials were more than a little displaced from each other.

To investigate this idea, pairs of test data sets and base data sets were compared and a weighting factor and point estimate of reliability was determined using the TRW method and the computer routines written for this project. A range of probabilities of success on a single trial and total number of trials was considered for both base and test data sets. Thirty six pairs were considered.

Table 2 is a tabulation of the data from the investigation. The number of trials in the base set, $n_{\rm b}$, and in the trial data set, $n_{\rm t}$, are shown for each case as are the underlying probabilities of success on a single trial, $P_{\rm b}$ for the base data and $P_{\rm t}$ for the trial data. Fach of the ten replications using these parameters resulted in a number of successes, $s_{\rm b}$ and $s_{\rm t}$, and point estimates of reliability, $P_{\rm b}$ and $P_{\rm t}$. These last were calculated as

$$P_b = \frac{s_b}{n_b}$$
 and $P_t = \frac{s_t}{n_t}$.

The number of successes and the number of failures in each replication were inputs into the computer program which generated the TRW weighting factor, K. The TRW point estimate



	ment to Base Data Point Estimate	$ R-P_b $.0229	.0297	.0296	.0297	.0296	.0229	.0107	.0151	.0140	.0059	.0146	.0149	.0100
	Deviation) Average Adjust-		(.0149)	(.0579)	.1378)	(00100)	(.0200)	(.0625)	(7000)	(.0044)	(:0303)	(.0267)	(.0256)	(.0385)	(*090*)
Relia- Estimate	Average TRW Method Relia- bility Fstimate (with Standard	l¤	.8771 (,8597	, 7896	,9153 (.9154 (.8429 (.9493 (.9449	9380 (,9181 (.9334 () 6016.	.8220 (
	Average TRW Method Weight- ing Factor	l⊠	.5971	.4970	.3977	.4970	.3977	.5971	.5595	.1868	.1041	.7782	.1351	.3048	.0953
sa Set	Average Point Estimate of Reliability	اما د ب	.82	.945	.945	. 83	.76	06.	.922	.822	.780	906.	.78	86.	86.
Test Data	Average No. of Successes	10,	16.4	18.9	18.9	8.3	7.6	0.6	46.1	41.1	39.0	45.3	39.0	49.0	49.0
Пе	Single Trial Prob of Success	μ t	. 80	66.	66.	06.	.80	66.	06.	.80	. 80	06.	.80	66.	99.
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Sets	Average Point Estimate of reliability	PI Q	06.	. 83	.76	.945	.945	.82	96.	96.	.952	.924	.948	968.	.812
ata	Average No. of Successes	I &	0.6	8.3	7.6	18.9	18.9	16.4	24.0	24.0	23.8	23.1	23.7	22.4	20.3
Base D	Single Trial Prob. of Success	P	66.	06.	. 80	66.	66.	. 80	66.	66.	66.	.95	.95	06.	. 80
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	Case Number		Н	7	m	Ą	5	9	7	∞	0	10	11	12	13



$\frac{R-P_{b}}{b}$	6600.	.0144	.0149	.0100	6600.	.0144	.0107	.0059	.0153	.0140	.0146	9010.	.0023	.0107	.0081	.0028	.0012	.0028	.0012	9900.	6600.	.0023	.0081	
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lw th	48.8	47.9	22.4	20.3	19.8	19.4	24.0	23.1	24.0	23.8	23.7	89.5	0.08	6.68	0.08	7.86	0.96	41.9	39.05	48.8	47.3	48.8	47.3	e 5
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lw Q	19.8	19.4	49.0	49.0	48.8	47.9	46.1	45.3	41.1	39.0	39.0	48.8	48.8	47.3	47.3	41.9	39.05	98.7	0.96	89.5	6.68	80.0	80.0	
P _D	.80	.80	66.	66.	66.	.95	06.	06.	08.	08.	08.	66.	66.	.95	.95	.80	08.	66.	.95	06.	06.	08.	.80	
n O	25	25	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20	100	100	100	100	100	100	
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of reliability, R, was then calculated as

$$R = \frac{1.0 \times P_b' + K \times P_t'}{1.0 + K}.$$

The magnitude of the adjustment to the base data point estimate is the absolute value of the difference $R-P_{\hat{b}}$. The values generated in the ten replications were averaged and these average values, \overline{s}_b , $\overline{P}_{\hat{b}}$, \overline{s}_t , \overline{P}_t , \overline{K} , \overline{R} , and $|\overline{R-P_{\hat{b}}}|$, are the entries in table 2.

By way of example, consider case 1. In this case, the base data set consisted of ten trials (n_b) , each with a probability of success of .99 (P_b) . The test data set consisted of twenty trials (n_t) , each with a probability of success of .80 (P_t) . The results of each of the ten replications are shown below in table 3. (Data from all replications is contained in the computer output section of this report.)

Replication	sb	Р́р	s _t	Ρí	TRW K	R	R-P'b
1	9	.90	16	.80	.5024	.8666	.0334
2	9	.90	16	.80	.5024	.8666	.0334
3	9	.90	17	.85	.6641	.8800	.0200
4	9	.90	17	.85	.6641	.8800	.0200
5	9	.90	17	.85	.6641	.8800	.0200
6	9	.90	17	.85	.6641	.8800	.0200
7	9	.90	18	.90	.8673	.9000	.0000
8	9	.90	18	.90	.8673	.9000	.0000
9	9	.90	13	.65	.2019	.8580	.0420
10	9	.90	15	.75	.3731	.8592	.0408
Averages	9.0	.90	16.4	.82	.5971	.8771	.0229

Table 3. Example showing the generation of data for Table 2, Case 1; Base Data Set: $n_b = 10$, $P_b = .99$, Test Data Set: $n_t = 20$, $P_t = .80$.

The last row in table 3 shows the average figures that are the table 2 entries for case 1. Table 2 also lists the standard deviation of the distribution of R for each case.



The data of table 2 shows that the TRW method consistently makes a correction to the point estimate of reliability that is in the proper direction. The largest correction to the point estimate in any of the 360 computations was .042. This occurred when the base data was nine successes out of ten trials and the test data was thirteen successes out of twenty trials. As hypothesized, as the number of trials in either data set increased, or as the point estimates from the data sets diverged, the correction to the point estimate decreased. Whenever the point estimates coincided, there was no correction.



IV. CONCLUSIONS

The correction that the TRW method applies to a base data point estimate decreases as the number of trials in either data set increases or as the point estimates from the data sets diverge. The method probably would adjust the point estimate less than would an observer making a subjective judgement based on the data.



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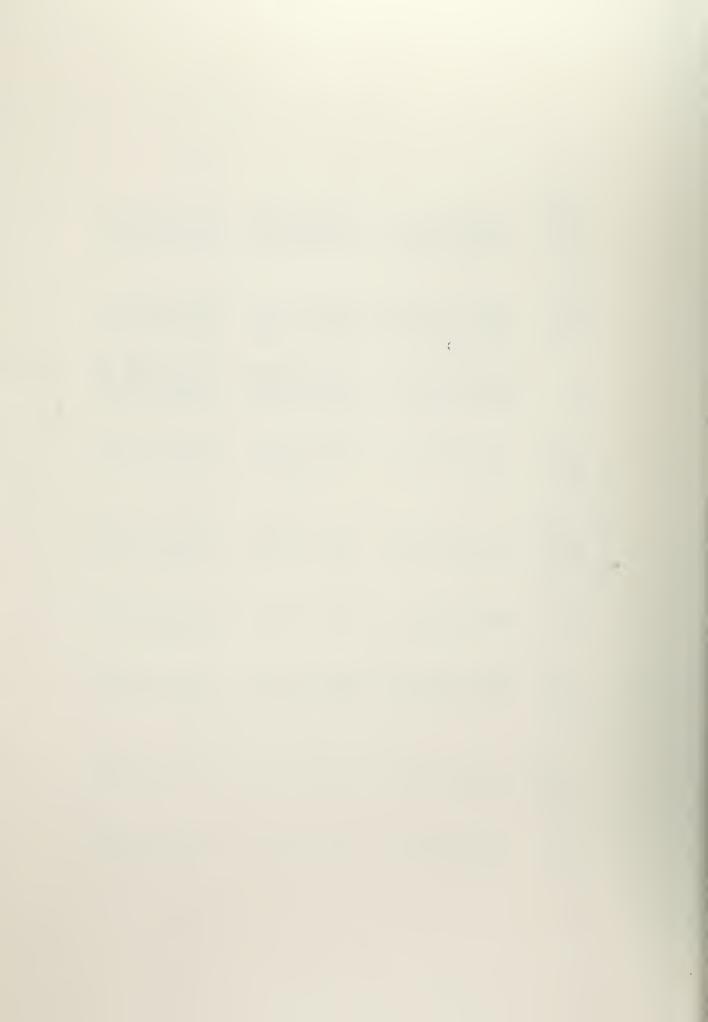
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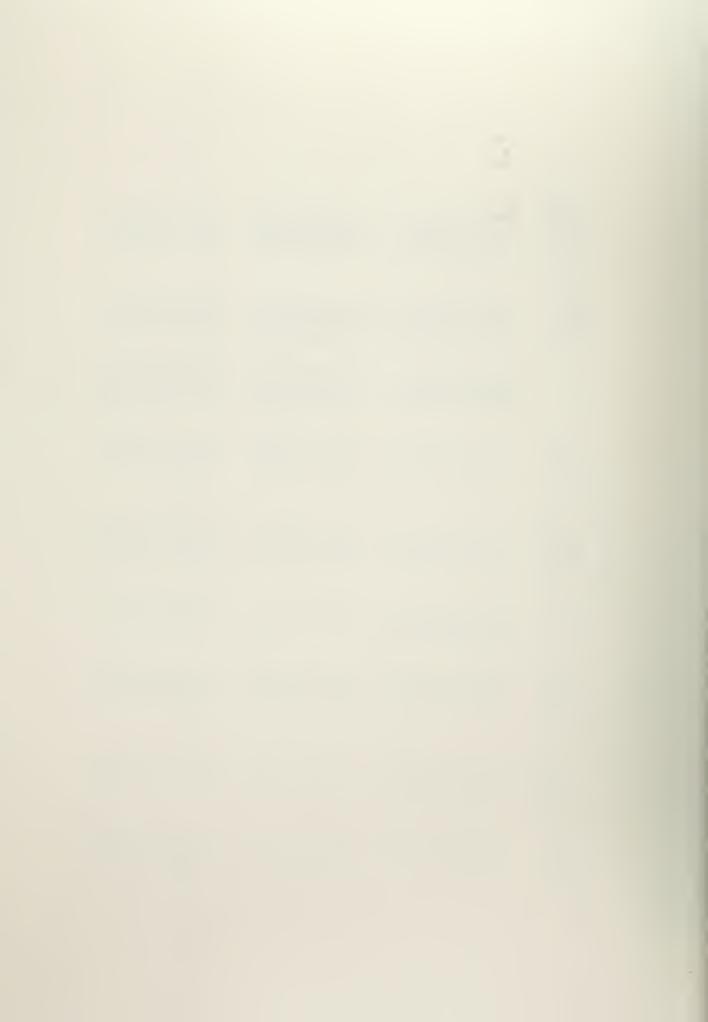
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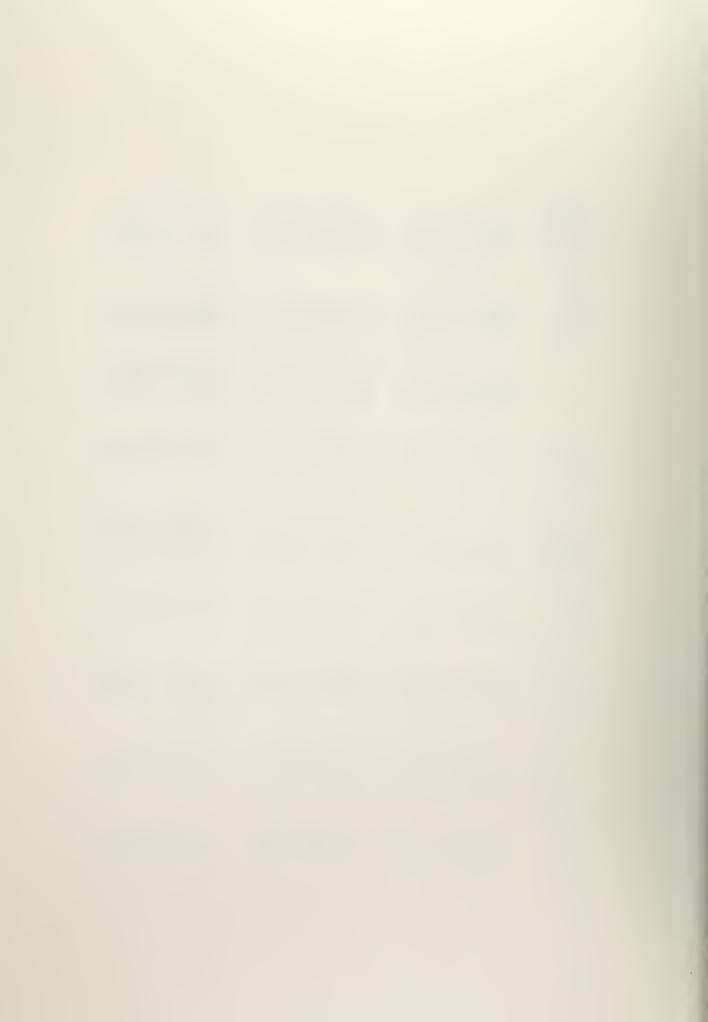
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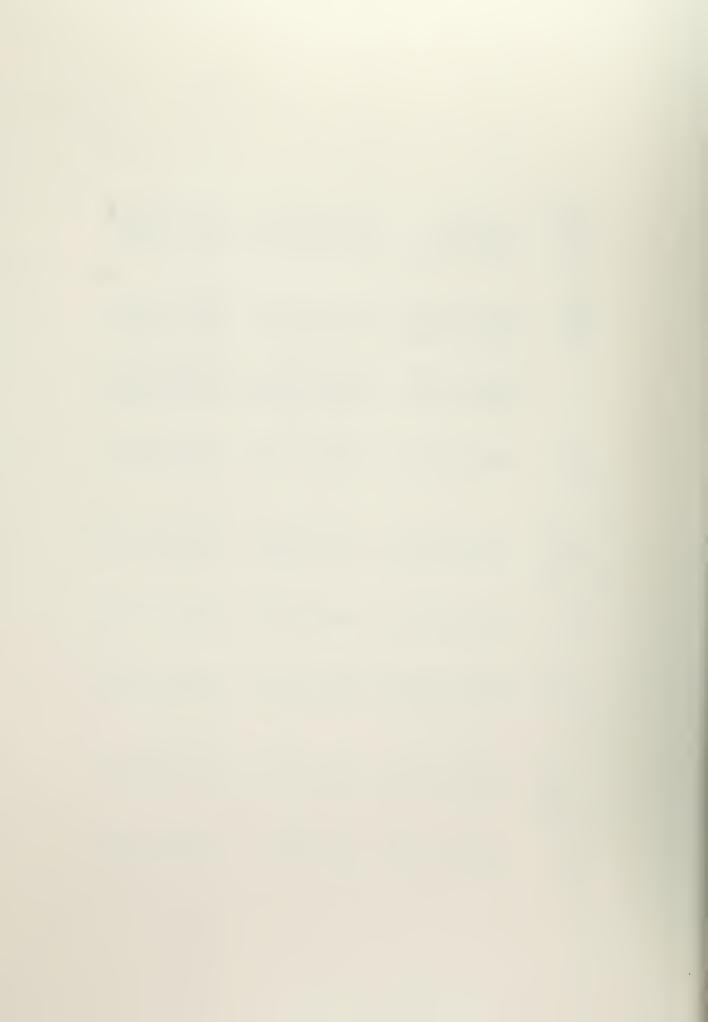
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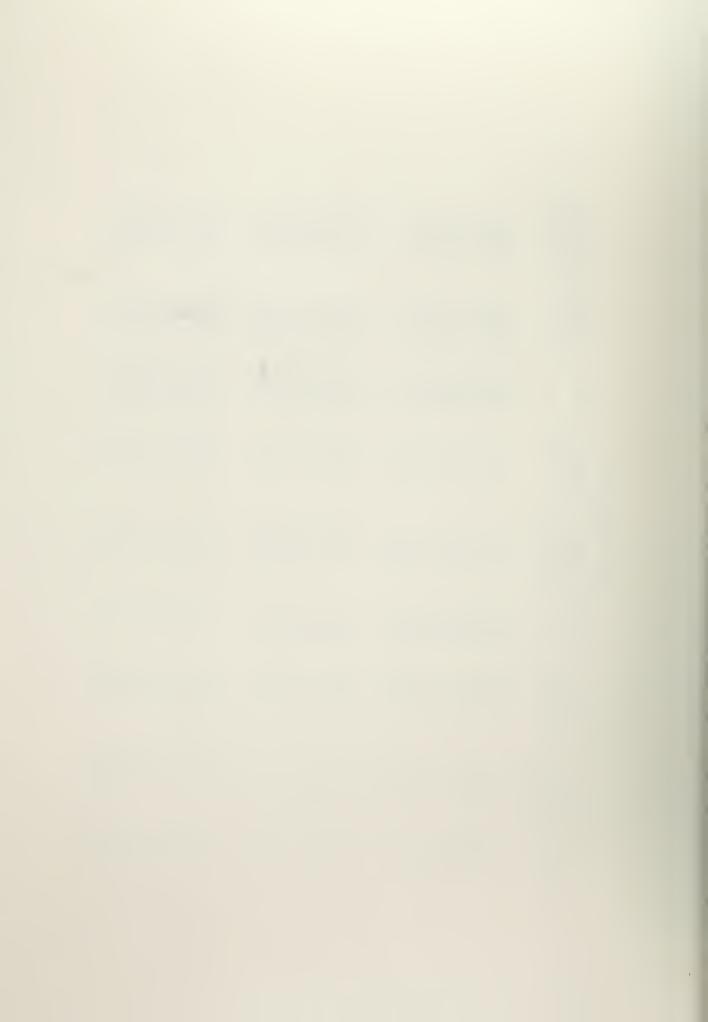
	• = = = = = = = = = = = = = = = = = = =			
	ADJUST TO BAS POINT ESTIMA	00000000000000000000000000000000000000	00178 00178 00178 00183 00197	00000000000000000000000000000000000000
	ESTIMATE OF REL	994655 994655 994655 99417 99600 99600	94422 94422 94422 94422 94422 94422 94422 94422 94422 94423 94465	9200 9200 9467 9467 9467 8399 9299 9520
	T K W K	0000 0000 0000 0000 0000 0000 0000 0000 0000	444 11100 1000	\$88 \$7,40000 \$7,40000 \$7,40000 \$7,400 \$1,400
A SET	ESTINATE OF REL	00000000000000000000000000000000000000	2008 2007 2008 2007 2007 2007	00000000000000000000000000000000000000
TEST DATA	SINGLE FRIAL PROBOGESS	000000000000000000000000000000000000000	00000000000000000000000000000000000000	00000000000
The company of the contract of	TRIALS	000000000	000000000	000000000
SET	POINT ESTIMATE OE. REL.	96 96 96 96 97 97 97 97 97 97 97 97 97	96 97 97 97 97 97 97 97 97	922 922 938 945 952 953
SE DATA	SINCLE FAILL PROBLESS	\$0\$0\$0\$0\$0\$0\$ \$0\$0\$0\$0\$0\$0\$ \$0\$0\$0\$0\$0\$		22 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2
ВА	TRIALS	10000000000 でいかなからいいからい	できないないできない。 できないないない。 できないないない。	とうという メンクシン



	ADJUST. TO BASE POINT ESTIMATE	00088 001288 00158 00056 00034	00.00000000000000000000000000000000000	00000000000000000000000000000000000000
	ESTIMATE OF REL	88088 80088 742528 87238 8649 8649	9633 96335 96335 76335 7632 7722 175	9672 9672 9672 9672 9742 9712 97112 9635
	TKW K	0516 00516 10008 005262 00131 00131	337.933 37.933 37.933 100.05 100.00 100.00 100.00	00000000000000000000000000000000000000
SET	ESTIMATE	\$0.0.0.0.0.0.0.0.0.0.0.0.0.0.0.0.0.0.0.	0.000 000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.000 0.	8884 444 77 70 880 77 77
TEST DATA	SINGLE TRIAL PROB. OF SUCCESS	555 555 555 555 555 555 555 555 555 55	220202020	220020000 220020000
	TRIALS	VNNNNNNNNNNNNNNNNNNNNNNNNNNNNNNNNNNNNN	プログラフグラウン ファックラファック ファックラック	~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~
SET	ESTIMATE OF REL	880 884 884 772 775 844 844 844 844	20000000000000000000000000000000000000	\$15000000000000000000000000000000000000
SE DATA	STRUCLE TRI AL PROS. OF SUCCESS	000000000000000000000000000000000000000	75555555555555555555555555555555555555	50 3 3 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5
B.A.	NR. OF TRIALS	スククシャックショウ いいいりゃく マンショウ	~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~	WWWWWWWW COOCOCO



	ADJUST. TO BASE POINT ESTIMATE	00000000000000000000000000000000000000	00120 00110 001110 00092 00092 000886	00207 00244 000247 000331 000331 00556
e en a de primer de la companya del la companya de	ESTIMATE OF REL	9741 9715 9715 9715 9732 9749 9749 9356	9480 9780 9489 9770 9770 9708 9714 9713	994493 9944933 994493 99444 99444
	TRW K	0632 13382 13382 0820 0820 0491 7791 7791	17632 1632 1632 16303 16305 16305 1777 1774 1774	10000000000000000000000000000000000000
A SET	FSTIMATE OF REL	\$2000 \$200 \$200 \$200 \$200 \$200 \$200 \$20	\$\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\	844 775 775 877 870 870 870 870
TEST DATA	SUCCESS	2220000000	0730075665	20000000000000000000000000000000000000
	TRI ALS	000000000000000000000000000000000000000	000000000000000000000000000000000000000	20220000000 20200000000000000000000000
SET	ESTIMATE OF REL	000000000 0000000000000000000000000000	366 366 366 366 366 366 366 366 366 366	9900.300.00 900.00 900.00 900.00 900.00 900.00
SE UNTA	SINGEE TRIAL PROB.3F SUCCESS	20000000000000000000000000000000000000	シンシウ メ ヤッシンションションションションションションションションションションションションショ	90 62 522 790 22 22 22 790 22 22 22 22 720
Va	NRUF- TRIALS	02000000000000000000000000000000000000	0303000000 0303000000	######################################



complete or the complete compl	TO BASE POINT ESTIMATE	01147 01183 01106 00106 000388 002755 0164	002222 002222 0003322 0003333 001064 000300	00016 00016 00058 00021 00033 00033
man on a color or color come come a colorabilistic or color	ESTIMATE OF REL.	8147 77706 77706 77706 77888 78888 787475 78364	88247 788827 788827 788827 78748 70630 70630	9899 9899 98890 9887 98891 98891 9891
	T.K.w. K	110000 10000000 100000000 10000000 1000000	25.00 11.00 10.00	00000000000000000000000000000000000000
SET	ESTIMATE OF PEL.	\$2000000000000000000000000000000000000	26 20 20 20 20 20 20 20 20 20 20 20 20 20	24000000000000000000000000000000000000
TEST DATA	-STNGLE- TRIAL PROB.UF SUCCESS	76766767666 6676667666	000000000 0000000000000000000000000000	000000000
	ARIALS	222222222	2702002000	000000000
SET	ESTIMATE OF REL.	80 94 77 74 74 75 75 70 70 70 70 70	80 776 778 778 778 70 70	9 6 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9
SE DATA	STRIGLE - PRIGE OF SUCCESS	2022222222 2022222222		
8 A 8	NR. CF	NN 1 NN 2 NN 1 NN 1 OO O O O O O O O O O O O O O	00000000000000000000000000000000000000	0000000000



	ADJUST. TO BASE POINT ESTIMATE	00000000000000000000000000000000000000	00000000000000000000000000000000000000	00161 00007 00040 00058 00171 00171 00035
	ESTIMATE OF REL	93552 93552 94669 96673 96673 9567 9567 9567	8220 7708 7708 7504 7916 7911 8430 8430	8361 77540 77540 77540 78571 88571 87473 87473
	T RW K	00000000000000000000000000000000000000	0129 00037 00037 00037 00057 00222 0222	0.0000333 0.0000333 0.0000333 0.00001 0.00001 0.00001 0.00001
SET	ESTIMATE DF REL	76 72 76 76 76 74 78 84 84	\$\coc\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\	92 93 94 94 95 96 96
TEST DATA	SINGLE TRIAL PROS. DE SUCCESS	######################################	9695996996 9695996996	000000000 0000000000000000000000000000
	VR. UF	00000000000000000000000000000000000000	00000000000	20000000000000000000000000000000000000
SET	POINT ESTIMATE CE REL	25.4 25.6 25.7 25.0 25.0 25.0 25.0 25.0 25.0 25.0 25.0	82777777777777777777777777777777777777	22 22 22 23 24 25 25 25 25 25 25 25 25 25 25 25 25 25
AS E DATA	SINGLE TRIAL PROB. OF SUCCESS	20 20 20 20 20 2 20 20 20 20 20 20 20 20 20 20 20 20 20 2	20000000000000000000000000000000000000	222222222 222222222
18	TRIALS	0000000000	0000000000	000000000000000000000000000000000000000



```
PROGRAM ONE
```

.

PURPOSE SOLVES FOR WEIGHTING FACTOR USED IN TRW METHOD OF CALCULATING OVERALL RELIABILITY. OUTPUT IS THE POINTS WHERE TWO BETA DENSITIES INTERSECT, THE AREA UNDEREACH CURVE FROM ORIGIN TO THOSE POINTS (CALLED P(X)), THE HEIGHT OR DENSITY AT THOSE POINTS, AND THE WEIGHTING OR K FACTOR.

METHOD

RTPLBS IS USED TO SOLVE THE POLYNOMIAL

X ** (A-C)* (1-X)** (B-D) = Y

WHERE

Y = ((C+D-1) *(A-1) *(B-1))/((A+B-1) *(C-1) *(D-1))

IF A-C AND B-D ARE BOTH NEGATIVE THEN THE POLYNOMIAL

X ** (C-A) * (1-X) ** (D-B) = 1/Y

SHOULD BE SUBSTITUTED. IF COIS NEGATIVE USE PROGRAM TWO. IF ONLY ONE OF THESE TERMS

SOLUTIONS BETWEEN O AND 1 ARE INPUTS FOR BOTR WHICH, WITH SUBSEQUENT PROGRAM LOGIC, SOLVES FOR WEIGHTING FACTOR.

PARAMETERS

COEF-A VECTOR OF COEFICIENTS OF POLYNOMIAL. MUST

BE DECLARED AS REAL®S OF DIMENSION N+1.

A -NUMBER OF SUCCESSES IN BASE DATA.

B -NUMBER OF FAILURES IN BASE DATA.

C -NUMBER OF SUCCESSES IN TEST DATA.

D -NUMBER OF FAILURES IN TEST DATA.

N -DEGREE OF POLYNOMIAL USED TO DETERMINE POINTS

OF INTERSECTION.

Y -CONSTANT TERM IN POLYNOMIAL.

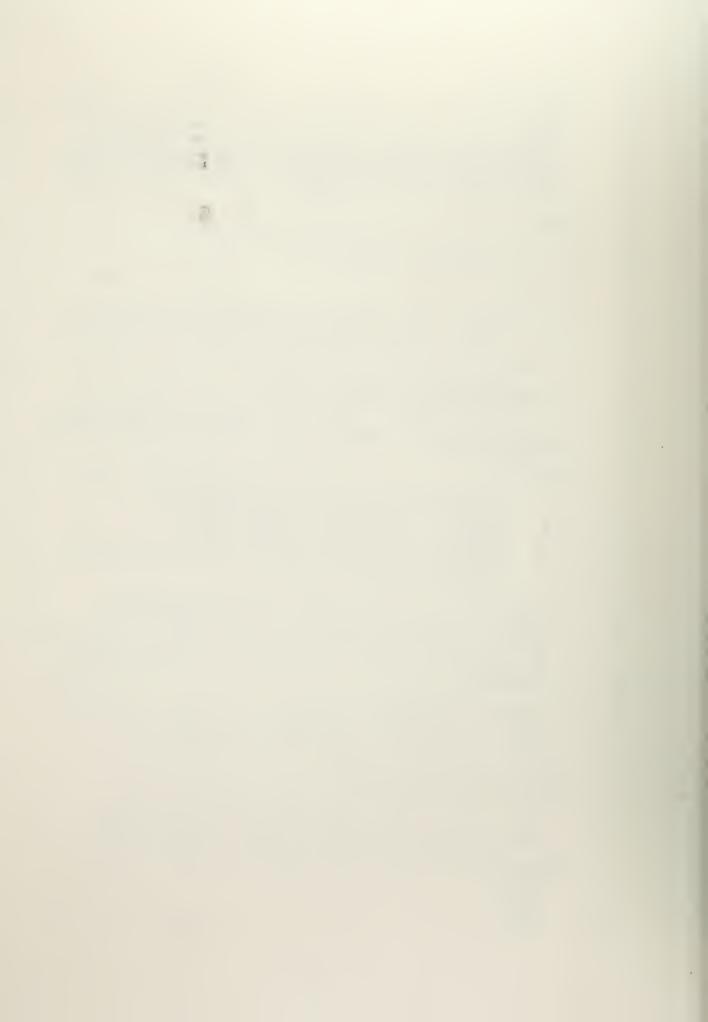
THE TERMS OF THE POLYHOMIAL MUST BE WORKED OUT MANUALLY AND MUST BE SUPPLIED THROUGH VECTOR COEF. Y MUST BE SUPPLIED AS FIRST STEP OF THE PROGRAM. MOST OF THE TERMS OF Y CANCELL. AN ARITHMETIC EXPRESSION CAN BE SUPPLIED TO COMPLETE THE CALCULATIONS. AFTER Y IS ENTERED THE PROGRAM SETS COEF(N+1) = Y.

INSERT DIMENSION OF VECTOR COEF AS N+1 IN TYPE STATE MENT AT LINE 0007 OF PROGRAM.
INSERT PARAMETERS A,B,C,D,N, AND VECTOR COEF IN DATA STATEMENT AT LINE 0008 OF PROGRAM.
INSEPT PARAMETER Y AT LINE 0009 OF PROGRAM.

LIMITS ON USE OF PROGRAM N MUST BE LESS THAN 49 EXPONENTS OF POLYNOMIAL MUST NOT BE NEGATIVE

IF EITHER OF THESE LIMITATIONS PROGRAM TWO AND CP-67/CMS TIME ARE NOT SHARING MET SYSTEM.

SUBROUTINES REQUIRED TO BE SUPPLIED BY USER RTPLSB BOTR DEGAM NOTP CDTR



```
CALL ERRSET (207,300,-1,1)

1))) FORMAT('1',' COMPUTED CONSTANT IS ',D13.6)

1100 FORMAT('0',5X,'ROOT IS ',D23.6)

1200 FORMAT('0',5X,3D23.6)

1700 FORMAT('0',5X,3D23.6)

1700 FORMAT('0',5X,1FOR X= ',E13.6,' A BETA(',F5.1,',', 2F5.1,') HAS P(X) ',F10.7, AND DENSITY ',F10.7

3/1X,'THE ERROR CODE IS ',I4//)

1800 FORMAT ('0', ' K FACTOR IS ', F10.7)

0007 REAL*8 COEF( ),U(49),V(49),CONV(49),Y,ROOT(5),

2DEN1(2),DEN2(2),PROB1(2),PROB2(2)

0008 DATA COEF/
2U,V,CONV/49*0.0,49*0.0/,ROOT/5*0.0/,

3PROB1/2*0.0/,PROB2/0.0/,DEN1/2*0.0/,DEN2/2*0.0/,N/ /

4,A,B,C,D/

0009 Y =
    Y = -Y
    WRITE (6,1000) Y
                        Y = -Y
WRITE (6,1000) Y
M = N + 1
                        COEF(M) = Y
                        CALL RIPLSB TO GET ROOTS AND DETERMINE REAL ROOTS
                     CALL RTPLSB (N,COEF,U,V,CONV, IER)
DO 25 J=1,N
IF (V(J)) 25,21,25
IF (U(J)) 25,25,26
IF (U(J) - 1.0) 27,27,25
ROOT (I) = U(J)
I = I + 1
CONTINUE
IF (ROOT(1)-ROOT(2)) 30,30,31
TEMP=RCOT(1)
ROOT(1)=ROOT(2)
ROOT(2)=TEMP
II = I - 1
DO 50 K =1,2
            21
26
27
             31
            30
           CALL BOTR (ROOT(K), A, B, PROB1(K), DEN1(K), IER)
WRITE (6,1700) ROOT(K), A, B, PROB1(K), DEN1(K), IER
CALL BOTR(ROOT(K), C, D, PROB2(K), DEN2(K), IER)
WRITE (6,1700) ROOT(K), C, D, PROB2(K), DEN2(K), IER
50 CONTINUE
C
                        IF (PROB1(1)-PRO32(1)) 52,54,53
Z=PROB1(1)+PROB2(2)-PROB2(1)+1.0-PROB1(2)
                         GO TO 60
                        IF (PROB1(2) - PROB2(2)) 53,53,52
Z=PROB2(1)+PRO31(2)-PROB1(1)+1.0-PROB2(2)
WRITE (6,1800) Z
            54
53
                        WRITE
END
            60
```



```
PROGRAM TWO CP-67/CMS NAME FOR PROGRAM IS EVAL2
                              PURPOSE
WHEN PROGRAM ONE IS NOT USED TO SOLVE FOR WEIGHTING FACTOR, THIS SIMPLE LOOK UP TYPE PROGRAM CAN BE USED WITH CP-67/CMS TO CONVERGE TO POINTS OF INTER SECTION OF TWO BETA CURVES AND TO GET AREAS UNDER THE CURVES FROM THE ORIGIN TO THE POINTS OF INTERSECTION. CALCULATION OF THE WEIGHTING FACTOR CAN THEN BE DONE BY HAND AFTER OBSERVING WHICH CURVE IS LOWER TO THE LEFT OF THE INTERSECTION.
                               METHOD
THE
                                                  HOD
THE PARAMETERS FOR TWO BETA DISTRIBUTIONS ARE READ
IN AS INPUTS. ALSO INPUTS BY A READ STATEMENT
SUPPLIED TO SUBROUTINE BOTR WHICH SUPPLIES AS OUTP
THE DENSITIES OR HEIGHTS OF THE TWO CURVES AT THE
INPUT POINTS. THE USER CAN OBSERVE CONVERGENCE
ON INTERSECTIONS (PCINTS OF EQUAL HEIGHT ON BOTH
CURVES). HE CAN THEN READ IN NEW VALUES FOR INPUT
POINTS UNTIL HE LOCATES THE INTERSECTIONS. THIS C
BE DONE QUITE RAPIDLY WITH TIME SHARING.
                                                                                                                                                                                                                                                                                                                                                 DUTPUT
                                                                                                                                                                                                                                                                                                                                      THIS CAN
                                PARAMETERS
A-NUMBER
B-NUMBER
                                                                                                                          SUCCESSES
FAILURES
SUCCESSES
FAILURES
AT WHICH B
                                                                                                                                                                                         IN BASE DATA.
IN BASE DATA.
IN TEST DATA.
IN TEST DATA.
ETA CURVES ARE
                                                                                                          OF
OF
                                                                                                                                                                                  IN B
                                                  C-NUMBER OF SUD-NUMBER OF FAS-ORDINATE AT T-ORDINATE AT V-ORDINATE AT W-ORDINATE AT W-ORDINATE AT W-ORDINATE AT
                                                                                                                                                                                    IN
                                                                                                                                                                                                                                                                                  EVALUATED.
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                                                                                                                                                                              BETA
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BETA
BETA
                                                   X-ORDINATE
Y-ORDINATE
Z-ORDINATE
                                                                                                                       AT
                                                                                                                                                                                 BETA
                                                                                                                        AT
                                                                                                                       AT
                                                                                                                                                                                 BETA
                                SUBROUTINES REQUIRED TO BE SUPPLIED BY USER BOTR DLGAM NOTR
                                                    CDTR
COFFLINE READ EVAL2 FORTRAN
800 FORMAT( 8F4.3)
900 FORMAT( 4F4.0)
1700 FORMAT( 1X, FOR X= ',E13.2)
2F5.1,' ) HAS P(X) '
3,14)
READ(5,900) A,8,C,F
READ(5,300)S,T,U,V,W ,X,Y,Z
CALL BDTR(S,A,B,P,D,IER)
WRITE(6,1700) S,A,B,P,D,IER
CALL BDTR(T,A,B,P,D,IER)
WRITE(6,1700) T,A,B,P,D,IER
CALL BDTR(T,A,B,P,D,IER)
WRITE(6,1700) T,A,B,P,D,IER
CALL BDTR(T,C,F,P,D,IER)
WRITE(6,1700) T,C,F,P,D,IER
WRITE(6,1700) U,A,B,P,D,IER
CALL BDTR(U,A,B,P,D,IER)
WRITE(6,1700) U,A,B,P,D,IER
CALL BDTR(U,C,F,P,D,IER)
WRITE(6,1700) U,C,F,P,D,IER
CALL BDTR(V,A,B,P,D,IER)
WRITE(6,1700) V,A,B,P,D,IER
CALL BDTR(V,A,B,P,D,IER)
WRITE(6,1700) V,A,B,P,D,IER
CALL BDTR(V,A,B,P,D,IER)
WRITE(6,1700) V,A,B,P,D,IER
CALL BDTR(V,C,F,P,D,IER)
WRITE(6,1700) V,A,B,P,D,IER
                                                                                                                                                                                 ',E13.6,' A BETA(',F5.1,',',',F10.7,' AND DENSITY ',F13.7
```



WRITE(6,1700) W,A,B,P,D,IER
CALL BDTR(W,C,F,P,D,IER)
WRITE(6,1700) W,C,F,P,D,IER
CALL BDTR(X,A,B,P,D,IER)
WRITE(6,1700) X,A,B,P,D,IER
CALL BDTR(X,C,F,P,D,IER)
WRITE(6,1700) X,C,F,P,D,IER)
WRITE(6,1700) Y,A,B,P,D,IER)
WRITE(6,1700) Y,A,B,P,D,IER)
WRITE(6,1700) Y,C,F,P,D,IER)
WRITE(6,1700) Y,C,F,P,D,IER)
WRITE(6,1700) Z,C,F,P,D,IER
CALL BDTR(Z,A,B,P,D,IER)
WRITE(6,1700) Z,A,B,P,D,IER
CALL BDTR(Z,C,F,P,D,IER)
WRITE(6,1700) Z,C,F,P,D,IER



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 March 1971.



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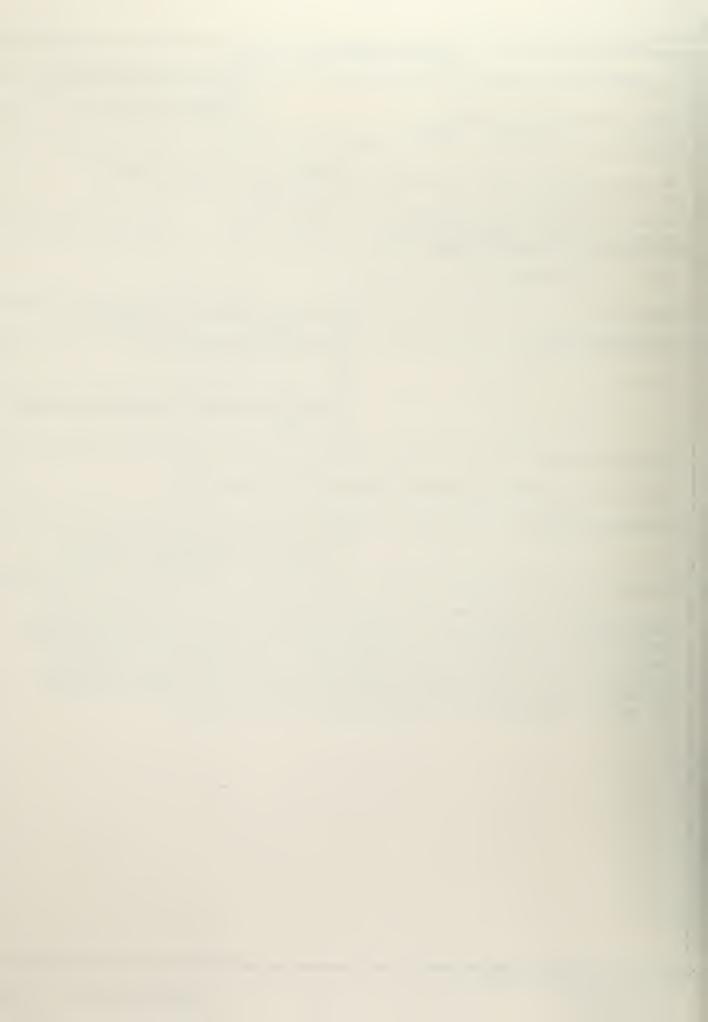


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11. SUPPLEMENTARY NOTES 12. SPONSORING MILITARY ACTIVITY Naval Postgraduate School Monterey, California

13. ABSTRACT

A computer program was written that carries out reliability assessments according to a method proposed by the TRW Corporation. This method combines point estimates for reliability from different sources into an overall point estimate. The program was used to calculate the overall point estimate for cases covering a range of sample sizes, and underlying probabilities of success in order to make a judgement on the usefulness of the method.



Security Classification LINK A LINK B LINK C KEY WORDS ROLE ROLE ROLE Reliability Point Estimates







Thesis
S4463 Sheridan
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TRW method of combining
point estimates of reliability.

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